## Problems Collection (Volume X)

These ten problems come from Chinese National Olympiad in Mathematics - Province Contest.
Problem 1 Polynomial $P(x)=x^{5}+a_{1} x^{4}+a_{2} x^{3}+a_{3} x^{2}+a_{4} x+a_{5}$, and we know when $k=1,2,3,4$, $P(k)=2007 *$. Calculate $P(10)-P(-5)$.

Problem 2 The sum of 100 positive integers $a_{1}, a_{2}, \ldots, a_{100}$ is 2007. Calculate the maximum possible value of $\left\lfloor^{\lll j<k \leq 100} a_{i} a_{j} a_{k}\right.$.

Problem 3 Calculate 100101102103104...... 498499500 modulo 126.
Problem 4 We define the sum of the first $n$ numbers of geometric progression $\left\{a_{n}\right\} S_{n}$. Now we know $\mathrm{S}_{7}=7, \mathrm{~S}_{14}=2014$. Calculate $\mathrm{S}_{7}{ }^{\star}\left(\mathrm{S}_{21}-\mathrm{S}_{14}\right)$.

Problem 5 Calculate the sum of this kind of positive integers $n(n>=4)$ : $n$ satisfies that $n$ ! can be written as the product of $n-3$ consecutive positive integers.

Problem 6 Two vertices of a square are on the line $y=2 x-17$, while the other two are on the parabola $y=x^{2}$. Calculate the sum of two different possible values of the area of this square.

Problem 7 A, B, C, D are four fixed points in the space and they are not on the same plane. Calculate the number of different parallelepipeds, which satisfies that 4 vertices of the parallelepiped are $A, B, C$ and $D$.

Problem 8 Polynomial $x^{2}-x-1$ exactly divides Polynomial $a_{1} x^{17}+a_{2} x^{16}+1$. Calculate $a_{1}{ }^{*} a_{2}$.
Problem 9 Suppose $x$ is an acute angle, calculate the minimum possible value of $(\sin x+\cos$ $\mathrm{x}) /(\sin \mathrm{x}+\tan \mathrm{x})+(\tan \mathrm{x}+\cot \mathrm{x}) /(\cos \mathrm{x}+\tan \mathrm{x})+(\sin \mathrm{x}+\cos \mathrm{x}) /(\cos \mathrm{x}+\cot \mathrm{x})+(\tan \mathrm{x}+\cot \mathrm{x}) /(\sin \mathrm{x}$ $+\cot \mathrm{x})$.

Problem 10 Suppose $x^{4}+y^{4}+z^{4}=m / n, x, y, z$ are all real numbers, satisfying $x^{*} y+y^{*} z+z^{*} x=1$ and $5^{*}$ $(x+1 / x)=12^{*}(y+1 / y)=13^{*}(z+1 / z) ; m$, $n$ are positive integers and their greatest common divisor is 1 . Calculate $\mathrm{m}+\mathrm{n}$.

## Input

There is no input.

## Output

Ten lines, each contains a single integer denoted the answer to the correspoding problem.

## Example

There is no example.

